The Value of Variance

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Outline

• “Theory”
  – National Institute of Standards and Technology (NIST) guidelines
    Law of propagation of uncertainty
  – ADJUSTED SPEEDUP EQUATION

• Experimental Setup
  hardware
  runtimes
  algorithms and input

• Two comments about the algorithms

• Experimental results

• Summary and Conclusions
“THEORY”
National Institute of Standards and Technology (NIST) guidelines

- Consider a quantity being measured, $Y$, in terms of other quantities, $X_i$:
  
  $$ Y = f(X_1, X_2, \ldots, X_N) $$

- The estimate:
  
  $$ y = f(x_1, x_2, \ldots, x_N) $$

- Sample mean of inputs:
  
  $$ x_i = \bar{X}_i = \frac{1}{n} \sum_{k=1}^{n} X_{i,k} $$

- Standard deviation as uncertainty measurement
  
  $$ u(x_i) = u_i = \sigma_i = \left( \frac{1}{n(n-1)} \sum_{k=1}^{n} (X_{i,k} - \bar{X_i})^2 \right)^{\frac{1}{2}} $$
The law of propagation of uncertainty

• If multiple quantities $X_1, X_2, \ldots, X_N$ are involved in the calculation of estimate $y$, the combined standard uncertainty is the positive square root of the estimated variance $\sigma^2(y)$ obtained from

$$
\sigma^2(y) = \sum_{i=1}^{N} \left( \frac{\partial f}{\partial x_i} \right)^2 \sigma^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} \sigma(x_i, x_j).
$$
Adjusted Speedup Equation

• Consider average execution time of two algorithms A and B as $\bar{t}_A$ and $\bar{t}_B$, (with uncertainty)

• Speedup equation:

$$ S = \frac{\bar{t}_A}{\bar{t}_B} $$

• Proposed adjusted speedup equation:

$$ S_{adj} = S \pm \sigma_i $$

where

$$ \sigma_i^2 = \frac{1}{\bar{t}_B^2} \sigma_A^2 + \frac{\bar{t}_A^2}{\bar{t}_B^2} \sigma_B^2. $$
EXPERIMENTAL SETUP
Experimental Setup - hardware

• Experiments on Indiana University's BigRed 2 Cray XE6/XK7 supercomputer
  – two AMD Opteron 16-core x86 64 CPUs per node
  – 64 GB of RAM per node
  – Gemini interconnect

• 1 to 16 compute nodes for weak scaling (starting from scale 14)
  – 16 threads per node
Experimental Setup – runtime

• (hardware – BigRed2 CRAY )

• two different runtimes:
  – HPX-5 – under development
  – AM++ - less feature rich, but well optimized to balance quick work delivery vs. communication overhead
Experimental Setup – algorithm and data

- (hardware – BigRed2 CRAY)
- (runtime – HPX-5, AM++)

- two different distributed algorithms for SSSP problem
  - KLA with K=2
  - $\Delta$-stepping with $\Delta=1$
  - NOT OPTIMIZED FOR MAXIMUM PERFORMANCE!

- Input graph
  - With Graph500 RMAT generator
  - Maximum edge weight 255 and 100
  - Scale 14 (1 node) to scale 18 (16 nodes)
Experimental setup

• (hardware – BigRed2 CRAY)
• (runtime – HPX-5, AM++)
• (algorithms – KLA SSSP, $\Delta$-stepping SSSP)
• (input – Graph500)

• Experiments:
  – 5 runs for each scale
  – 8 problem instances per run (different starting point – source)
TWO COMMENTS ABOUT THE ALGORITHMS
Distributed Algorithms for SSSP

K-Level Asynchronous Algorithm

\[ K=1 \]

\[ \Delta = 1 \]

\[ S : \text{Source} \]

\[ d_u : \text{Distance from source to } u \]

\[ w_{uv} : \text{weight between } u \text{ and } v \]
Distributing work over nodes – Strangler Effect
EXPERIMENTAL RESULTS
Reporting Speedup Uncertainty: On HPX-5 Runtime

With max edge weight 255
Reporting Speedup Uncertainty: On HPX-5 Runtime (cont.)

With max edge weight 100
Reporting Speedup Uncertainty: On AM++ Runtime

With max edge weight 100
Relative Standard Uncertainty: On AM++ Runtime (execution times)

With max edge weight 100
Relative Standard Uncertainty: On HPX-5 Runtime (execution times)

With max edge weight 255

With max edge weight 100
SUMMARY AND CONCLUSIONS
Summary and Conclusions

• We looked at how standard deviation improves our insight from data
  – Simple to include
  – Additional insight that is lacking from aggregate measures (means)
  – Allows to drill down to what is interesting
  – Allows more meaningful comparison across different experiments

• Experiments using two distributed SSSP algorithms and two runtimes on BigRed2 CRAY

• DISCLAIMER – NOT about the algorithms and/or runtimes
  – During development of HPX-5; AM++ mature
  – Algorithm parameters not tuned

NOT INDICATIVE OF WHAT ALGORITHMS OR RUNTIMES CAN DO

• ALL ABOUT DATA and METHODOLOGY